Problema 9.3.4.

Utilizând metoda lui Quine simplificaţi următoarele funcţii booleene de trei variabile:

|  |  |
| --- | --- |
|  | 2. *f*2(*x*1*,x*2*,x*3)*=* *x*3(*x*1∨ *x*2) ∨ (¯*x*2↓*x*3); |

Identificarea suportului funcției (deci și a FCD):  
Tabela de valori:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *x*1 | *x*2 | *x*3 | *x*1∨ *x*2 | *x*3(*x*1∨ *x*2) | ¯*x*2 | ¯*x*2↓*x*3 | *f*2(*x*1*,x*2*,x*3) |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |

S*f* ={(0,1,0),(0,1,1),(1,0,1),(1,1,0),(1,1,1)}={(1,1,1), (1,1,0),(1,0,1),(0,1,1), (0,1,0)}

Tabelul de factorizare:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Grupul |  | *x*1 | *x*2 | *x*3 |  |
| *I* | *√* | 1 | 1 | 1 | *m*7 |
|  | *√* | 1 | 1 | 0 | *m*6 |
| *II* | *√* | 1 | 0 | 1 | *m*5 |
|  | *√* | 0 | 1 | 1 | *m*3 |
| *III*  *Factorizare simplă* | *√* | 0 | 1 | 0 | *m*2 |
| *IV=I+II* | *√* | 1 | 1 | - | *m*7∨ *m*6 |
|  | 1 | - | 1 | *m*7∨ *m*5= *max*1=*x*1*x*3 |
| *Factorizare dublă* | *√* | - | 1 | 1 | *m*7∨ *m*3 |
| *V*=*II*+*III* | *√* | - | 1 | 0 | *m*6∨ *m*2 |
|  | *√* | 0 | 1 | - | *m*3∨ *m*2 |
| *Nu avem Factorizare triplă* |  | - | 1 | - | (*m*7∨ *m*6) ∨( *m*3∨ *m*2) =(*m*7∨ *m*3) ∨( *m*6∨ *m*2) = *max*2= *x*2 |

M( *f* )={ *max*1, *max*2}

Tabelul de identificare a monoamelor centrale:

|  |  |  |
| --- | --- | --- |
| **mon.**  **max.**  **mintermi** | *max*1 | *max*2 |
| *m*7 | \* | \* |
| *m*6 |  | \* |
| *m*5 | \* |  |
| *m*3 |  | \* |
| *m*2 |  | \* |

C( *f* )={ *max*1, *max*2}

M( *f* )= C( *f* ) ⇒ cazul I al algoritmului de simplificare

*f*2’(*x*1, *x*2, *x*3)= *max*1∨ *max*2 = *x*1*x*3∨ *x*2

Problema 9.3.5.

Simplificaţi următoarele funcţii booleene de patru variabile date prin valorile de 1, utilizând metoda lui Quine:

1. *f*2(1,1,0,1)=1, *f*2(0,1,0,1)=1, *f*2(0,1,0,0)=1, *f*2(0,0,0,0)=1, *f*2(0,0,1,0)=1, *f*2(1,0,1,1)=1, *f*2(1,0,0,1)=1, *f*2(0,0,1,1)=1;

S*f* ={(0,0,0,0), (0,1,0,0),(0,0,1,0), (1,0,0,1),(0,0,1,1),(0,1,0,1), (1,1,0,1),(1,0,1,1)}

Tabelul de factorizare:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Grupul |  | ***x*1** | ***x*2** | ***x*3** | ***x*4** |  |
| *I* | *√* | 0 | 0 | 0 | 0 | *m*0 |
| *II* | *√* | 0 | 1 | 0 | 0 | *m*4 |
|  | *√* | 0 | 0 | 1 | 0 | *m*2 |
| *III* | *√* | 1 | 0 | 0 | 1 | *m*9 |
| *√* | 0 | 0 | 1 | 1 | *m*3 |
|  | *√* | 0 | 1 | 0 | 1 | *m*5 |
| *√* | 1 | 1 | 0 | 1 | *m*13 |
| *IV*  *Factorizare simplă* | *√* | 1 | 0 | 1 | 1 | *m*11 |
|  |  | 0 | - | 0 | 0 | *m*0∨ *m*4 = *max*1=¯ *x*1¯¯ *x*3¯*x*4 |
| *V*=*I*+*II* |  | 0 | 0 | - | 0 | *m*0∨ *m*2= *max*2=¯ *x*1¯¯ *x*2¯*x*4 |
|  |  | 0 | 1 | 0 | - | *m*4∨ *m*5= *max*3=¯ *x*1 *x*2¯*x*3 |
| *VI*=*II*+*III* |  | 0 | 0 | 1 | - | *m*2∨ *m*3= *max*4=¯ *x*1¯¯ *x*2*x*3 |
| *VII*=*III*+*IV* |  | - | 0 | 1 | 1 | *m*3∨ *m*11= *max*5=¯*x*2*x*3*x*4 |
|  | 1 | - | 0 | 1 | *m*9∨ *m*13= *max*6=*x*1¯¯ ¯*x*3*x*4 |
| *Nu avem Factorizare dublă* |  | 1 | 0 | - | 1 | *m*9∨ *m*11= *max*7=*x*1¯¯ *x*2*x*4 |
|  | - | 1 | 0 | 1 | *m*5∨ *m*13= *max*8=*x*2¯*x*3*x*4 |

M( *f* )={ *max*1, *max*2, *max*3, *max*4, *max*5, *max*6, *max*7, *max*8}

Tabelul de identificare a monoamelor centrale:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| mon.  max.  mintermi | *max*1 | *max*2 | *max*3 | *max*4 | *max*5 | *max*6 | *max*7 | *max*8 |
| *m*0 | \* | \* |  |  |  |  |  |  |
| *m*4 | \* |  | \* |  |  |  |  |  |
| *m*2 |  | \* |  | \* |  |  |  |  |
| *m*9 |  |  |  |  |  | \* | \* |  |
| *m*3 |  |  |  | \* | \* |  |  |  |
| *m*5 |  |  | \* |  |  |  |  | \* |
| *m*13 |  |  |  |  |  | \* |  | \* |
| *m*11 |  |  |  |  | \* |  | \* |  |

C( *f* )=∅

M( *f* )≠ C( *f* ), C( *f* ) =∅ ⇒ cazul III al algoritmului de simplificare

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| mon.  max.  mintermi | *max*1 | *max*2 | *max*3 | *max*4 | *max*5 | *max*6 | *max*7 | *max*8 |
| *m*0 | \* | \* |  |  |  |  |  |  |
| *m*4 | \* |  | \* |  |  |  |  |  |
| *m*2 |  | \* |  | \* |  |  |  |  |
| *m*9 |  |  |  |  |  | \* | \* |  |
| *m*3 |  |  |  | \* | \* |  |  |  |
| *m*5 |  |  | \* |  |  |  |  | \* |
| *m*13 |  |  |  |  |  | \* |  | \* |
| *m*11 |  |  |  |  | \* |  | \* |  |

*f*2’(1)(*x*1, *x*2, *x*3, *x*4)= *h*1(*x*1, *x*2, *x*3, *x*4)= *max*8∨ *max*7∨ *max*4∨ *max*1= *x*2¯*x*3*x*4∨ *x*1¯¯ *x*2*x*4∨¯ *x*1¯¯ *x*2*x*3∨¯ *x*1¯¯ *x*3¯*x*4

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| mon.  max.  mintermi | *max*1 | *max*2 | *max*3 | *max*4 | *max*5 | *max*6 | *max*7 | *max*8 |
| *m*0 | \* | \* |  |  |  |  |  |  |
| *m*4 | \* |  | \* |  |  |  |  |  |
| *m*2 |  | \* |  | \* |  |  |  |  |
| *m*9 |  |  |  |  |  | \* | \* |  |
| *m*3 |  |  |  | \* | \* |  |  |  |
| *m*5 |  |  | \* |  |  |  |  | \* |
| *m*13 |  |  |  |  |  | \* |  | \* |
| *m*11 |  |  |  |  | \* |  | \* |  |

*f*2’(2)(*x*1, *x*2, *x*3, *x*4)= *h*2(*x*1, *x*2, *x*3, *x*4)= *max*2∨ *max*3∨ *max*5∨ *max*6=¯ *x*1¯¯ *x*2¯*x*4∨¯ *x*1 *x*2¯*x*3∨¯*x*2*x*3*x*4∨ *x*1¯¯ ¯*x*3*x*4

*f*2’(2)(*x*1, *x*2, *x*3, *x*4)= *h*2(*x*1, *x*2, *x*3, *x*4)= *max*2∨ *max*3∨ *max*6∨ *max*7=¯ ¯*x*2¯*x*3 ¯ *x*4∨¯ *x*1¯¯ *x*3*x*4∨ *x*1*x*2¯ *x*4∨¯ *x*1*x*2*x*3

Problema 9.3.6.

Simplificaţi următoarele funcţii booleene de trei variabile date prin zerourile acestora, utilizând metoda lui Quine:

1. *f*2(0,0,0) = *f*2(0,0,1) = *f*2(1,1,1) = 0;

Identificarea suportului funcției:

S*f* ={(0,1,1),(1,0,1),(0,1,0),(1,0,0),(1,1,0)}={(0,1,1),(1,0,1),(1,1,0), (0,1,0),(1,0,0)}

Tabelul de factorizare:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Grupul |  | *x*1 | *x*2 | *x*3 |  |
| *I* | *√* | 0 | 1 | 1 | *m*3 |
|  | *√* | 1 | 0 | 1 | *m*5 |
|  | *√* | 1 | 1 | 0 | *m*6 |
| *II*  *Factorizare simplă* | *√* | 0 | 1 | 0 | *m*2 |
| *√* | 1 | 0 | 0 | *m*4 |
| *III= I*+*II* |  | 0 | 1 | - | *m*3∨ *m*2= *max*1=¯ *x*1*x*2 |
|  | 1 | 0 | - | *m*5∨ *m*4= *max*2=*x*1¯ *x*2 |
|  |  | - | 1 | 0 | *m*6∨ *m*2= *max*3=*x*2¯ *x*3 |
| *Nu avem Factorizare dublă* |  | 1 | - | 0 | *m*6∨ *m*4= *max*4=*x*1¯ *x*3 |

M( *f* )={ *max*1, *max*2, *max*3, *max*4}

Tabelul de identificare a monoamelor centrale:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| mon.  max.  mintermi | *max*1 | *max*2 | *max*3 | *max*4 |
| *m*3 | \* |  |  |  |
| *m*5 |  | \* |  |  |
| *m*6 |  |  | \* | \* |
| *m*2 | \* |  | \* |  |
| *m*4 |  | \* |  | \* |

C( *f* )={ *max*1, *max*2}

M( *f* )≠ C( *f* ), C( *f* ) ≠∅ ⇒ cazul II al algoritmului de simplificare

*g*(*x*1, *x*2, *x*3)= *max*1∨ *max*2

se hașurează coloanele cu steluțe încercute și apoi liniile cu steluțe hașurate

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| mon.  max.  mintermi | *max*1 | *max*2 | *max*3 | *max*4 |
| *m*3 | \* |  |  |  |
| *m*5 |  | \* |  |  |
| *m*6 |  |  | \* | \* |
| *m*2 | \* |  | \* |  |
| *m*4 |  | \* |  | \* |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| mon.  max.  mintermi | *max*1 | *max*2 | *max*3 | *max*4 |
| *m*3 | \* |  |  |  |
| *m*5 |  | \* |  |  |
| *m*6 |  |  | \* | \* |
| *m*2 | \* |  | \* |  |
| *m*4 |  | \* |  | \* |

*h1*(*x*1, *x*2, *x*3)= *max*3

*h2*(*x*1, *x*2, *x*3)= *max*4

Formele simplificate ale funcției sunt:

*f2* ‘(1)(*x*1, *x*2, *x*3)= *g*(*x*1, *x*2, *x*3)∨ *h1*(*x*1, *x*2, *x*3)= *max*1∨ *max*2∨ *max*3=¯ *x*1*x*2∨ *x*1¯ *x*2∨ *x*2¯ *x*3

*f2* ‘(2)(*x*1, *x*2, *x*3)= *g*(*x*1, *x*2, *x*3)∨ *h2*(*x*1, *x*2, *x*3)= *max*1∨ *max*2∨ *max*4=¯ *x*1*x*2∨ *x*1¯ *x*2∨ *x*1¯ *x*3

*g*(*x*1, *x*2, *x*3)= *max*1∨ *max*2 =¯ *x*2*x*3∨ *x*2¯*x*3

Formele simplificate:

*f*2’(1)(*x*1, *x*2, *x*3)= *g*(*x*1, *x*2, *x*3) ∨ *h*1(*x*1, *x*2, *x*3) = *max*1∨ *max*2 ∨ *max*3 =¯ *x*2*x*3∨ *x*2¯*x*3∨ *x*1*x*3

*f*2’(2)(*x*1, *x*2, *x*3)= *g*(*x*1, *x*2, *x*3) ∨ *h*2(*x*1, *x*2, *x*3) = *max*1∨ *max*2∨ *max*4 =¯ *x*2*x*3∨ *x*2¯*x*3∨ *x*1*x*2